A Numerical Model of the Urban Heat Island

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ABSTRACT

The heat island phenomenon is surveyed. Existing theories are criticized as being excessively qualitative. A general purpose, numerical energy budget model is described and applied to the urban atmosphere. Calculations for several special cases as well as a sensitivity analysis are presented. The model is found to predict the correct order of magnitude of the urban temperature excess. The heat island effect is found to be the net result of several competing physical processes. In general, reduced evaporation in the city center and the thermal properties of the city building and paving materials are the dominant parameters. It is suggested that such a model could be used in engineering calculations to improve the climate of existing and future cities.

1. Introduction

In this paper we describe an application of a numerical energy budget model to the urban heat island problem. The model, which was designed as a teaching aid for micrometeorology and ecology courses, has been found to have a utility beyond the specific application considered here. Consequently, we shall discuss the model in some detail in the third section of this paper before specializing to the urban atmosphere.

2. The urban heat island phenomenon

The existence of the urban heat island phenomenon has been documented for over a century (Howard, 1833). Howard's measurements showed the average urban temperature excess of London to be 2.0F, the excess being greatest at night, when it amounted to 3.7F. On the average, the heat island effect was largest in the winter and least in the spring. During the day, the situation was reversed with the city being 0.3F cooler than the surrounding countryside. These results are essentially the same as those of modern investigations (Chandler, 1962).

In recent years, heat island climatologies have been formed for many cities (Duckworth and Sandberg, 1954; Landsberg, 1956; Mitchell, 1961; Roden, 1966; Bornstein, 1968). We shall not review these studies in any detail since it is clear that the particular numbers are peculiar to each city. In general, the results are similar to those obtained by Howard. The nighttime excess is always the largest, while the daytime urban temperature is found to be less than the surrounding rural areas for some cities. The largest heat island was measured in San Francisco (Duckworth and Sandberg, 1954) where the temperature difference between a large park area and the city center was over 10C on one occasion. The

season of the maximum heat island varies. Unlike London, most cities develop the maximum temperature excess in the summer. Probability displays of the temperature excess show the effect is highly variable from day to day (Landsberg, 1956). Sundborg (1951) analyzed the heat island of Uppsala in terms of the prevailing meteorological conditions and found the wind speed to be a dominant parameter with decreasing temperature excess with increasing wind speed. Landsberg (1956) has shown that the magnitude of the heat island effect to be positively correlated with city size.

Many explanations have been given for the heat island. Howard believed that urban temperatures were raised by self-heating due to industrial and domestic combustion. Kratzer (1956) attributes the heat island primarily to the blanketing effect of urban atmospheric pollution. The pollution cloud at night is supposed to absorb and re-emit thermal radiation. from the city, resulting in the observed large nocturnal temperature excess. Other authors have pointed out the importance of reduced evaporation in the center of cities (Bornstein, 1968; Chandler, 1962). It is argued that if little evaporation or plant transpiration is occurring in the city center, then the available solar radiation must go into heating the city streets, buildings and air. Mitchell (1961) emphasizes the role of building and paving materials having large heat capacities and conductivities. The argument is that the city can absorb large amounts of heat, relative to rural soils, during the day which then becomes available at night to partially balance the nocturnal radiation loss. The effect of buildings on the wind field has been discussed in connection with the heat island. Mitchell reasons that buildings act to ". . . break up the cooling breezes that might otherwise offer relief." Chandler, on the other hand, suggests that buildings act to dissipate the temperature excess by augmenting

atmospheric turbulence which acts to transfer heat upward.

These observations and explanations suggest strongly that the urban heat island is the result of a complex set of interacting physical processes. In these circumstances any one-dimensional explanation can have little usefulness. It may be possible to identify the dominant parameter for the particular topography and meteorology of a given city, but it is decidedly misleading to generalize such results to other situations.

In surveying the explanations mentioned above, the complete absence of numerical estimates of the order of magnitude of the suggested mechanisms is striking. In this regard Landsberg refers to Sundorg's (1951) statistical analysis as ". . . an attempt to underbuild the observed facts by a theory." It is curious that pure empiricism appears to be "theoretical" in the literature of the heat island. In the next section, we will outline a simple energy budget theory to be applied to the urban atmosphere. In applying energy budget theory to the urban climate we will not consider the possible role of pollution on the radiation regime, although we note that this is undoubtedly an important aspect.

3. An atmospheric energy budget model

The energy budget model we discuss here was formulated as a teaching tool. Our objective was to form the simplest possible set of equations which still retained the essential physics of the atmospheric surface layer but which also could be conveniently solved with the use of a small computer in the classroom or laboratory.

This model is essentially derived from the work of Halstead *et al.* (1957). We start with the energy balance equation for the surface of the earth,

$$R_N = LE + H + S, \qquad (1)$$

where R_N is the net radiation flux, E the evaporation rate, L the latent heat of water (so that LE is the latent heat flux), H the sensible heat flux to the air, and S the flux of heat into the soil. The terms on the right are defined to be positive for transfer away from the interface. Our strategy is now to seek physical relationships for each of these terms in such a way that we obtain a closed set of equations. The net radiation term can be written as

$$R_N = (1-\alpha)T_r R_0 |\sin\phi \sin\delta + \cos\phi \cos\delta \cos\gamma| - IR_N, \qquad (2)$$

where α is the albedo for incoming solar radiation, T_r a transmission coefficient for the atmosphere, R_0 the solar constant, ϕ latitude, δ solar declination, γ the solar hour angle, and IR_N the net infrared flux at the surface of the earth.

The turbulent fluxes of latent and sensible heat may be written as

$$H = -\rho C_p K_h \frac{\partial \theta}{\partial Z}, \qquad (3)$$

$$LE = -\rho L K_v \frac{\partial q}{\partial Z}, \qquad (4)$$

where θ is potential temperature, ρ air density, C_p the specific heat at constant pressure, q the specific humidity, K_h and K_v the turbulent diffusivities for heat and water vapor, respectively, and Z distance from the interface. Eqs. (3) and (4) are only definitions of diffusivity. For neutral and near-neutral stability, the diffusivity for momentum, K_m, is given by

$$K_m = kZU_*, \qquad (5)$$

where k is the von Kármán constant, and U_* the friction velocity, defined by

$$U_* = (\tau/\rho)^{\frac{1}{2}}, \qquad (6)$$

where τ is the downward flux of momentum. Assuming τ is constant, the logarithmic wind law may be integrated to yield

$$U = \left(\frac{U_*}{k}\right) \ln\left(\frac{Z}{Z_0}\right), \qquad (7)$$

where Z_0 is a function of surface roughness. Solving this equation for U_* and substituting in (5) gives

$$K_m = \frac{k^2 U Z}{\ln(Z/Z_0)}. \qquad (8)$$

We now assume that $K_h = K_v = K_m$ so that the turbulent fluxes become:

$$H = \left[\frac{-\rho C_p k^2 U}{\ln(Z/Z_0)}\right] \frac{\partial \theta}{\partial \ln Z}, \qquad (9)$$

$$LE = \left[\frac{-\rho L k^2 U}{\ln(Z/Z_0)}\right] \frac{\partial q}{\partial \ln Z}. \qquad (10)$$

The soil heat flux is given by

$$S = -k_s \frac{\partial T}{\partial Z}, \qquad (11)$$

where T is soil temperature, k_s the soil thermal conductivity, and Z distance from the interface. In order to calculate the soil heat flux at any time, it is necessary to solve the one-dimensional form of the Fourier heat conduction equation, i.e.,

$$\frac{\partial T}{\partial t} = \frac{k_s}{\rho_s C} \frac{\partial^2 T}{\partial Z^2}, \qquad (12)$$

where ρ_s and C are soil density and specific heat capacity.

We lack as yet a boundary condition for water vapor. Rather than attempting to calculate soil water transport, we make the assumption that the specific humid-

ity q_0 at height Z_0, is a function of the temperature T_0 at that level. If the air is saturated, as it would be in a freely transpiring plant canopy, then

$$q_0 = q_{sat}(T_0), \qquad (13)$$

Where $q_{sat}(T_0)$ is the saturation specific humidity at the temperature T_0. The saturation specific humdtiiy function can be approximated by

$$q_{sat} = \frac{1}{L}[3.74 + 2.64(T_0/10)^2] \times 10^{-3}. \qquad (14)$$

In order to make calculations over an unsaturated canopy, such as a city, we add the definition of relative humidity, RH, to the above relationships and, after recombination, we have

$$q_0 = \frac{RH}{L}[3.74 + 2.64(T_0/10)^2] \times 10^{-3}, \qquad (15)$$

where we will take RH as constant in these calculations. Over a surface consisting of a mixture of freely transpiring canopies and totally dry streets and buildings, we will assume that RH can be interpreted as the fraction of total area occupied by transpiring plants.

Eqs. (1), (2), (9), (10), (11), (12) and (15) now constitute the model. To proceed further, the equations must be put in finite difference form. We define the following heights: Z_1 is the canopy height and Z_2 is a height well above the canopy, where we will assume the meteorological conditions to be constant. In the soil we will assume the temperature to be constant at depth $2d$, and we will calculate temperature only at depth d. We assume the air temperature to be isothermal in the canopy between the surface and height Z_0, so that T_0 can also be interpreted as the surface temperature.

In finite difference form equations (9), (10), (11), and (12) then become:

$$H = \left\{ \frac{-\rho C_p k^2 U_*}{[\ln(Z_2/Z_0)]^2} \right\} (T_2 + \Gamma_d Z_2 - T_0), \qquad (16)$$

$$LE = \left\{ \frac{-\rho L k^2 U_2}{[\ln(Z_2/Z_0)]^2} \right\} (q_2 - q_0), \qquad (17)$$

$$S = \frac{-k_s}{d}(T_s - T_0), \qquad (18)$$

$$T_s = \frac{k_s}{\rho_s C d^2} \int_0^t (T_b - 2T_s + T_0)dt, \qquad (19)$$

where Γ_d is the adiabatic lapse rate, T_s the soil temperature at depth d, T_b the soil temperature at depth $2d$, and the quantity in parentheses in (19) is the finite difference form of the second derivative in Eq. (12). It should be noted that we have assumed that H, LE and U_* are

constant between Z_0 and Z_2. This assumption is inconsistent with the earlier assumption that Z_2 is high enough that the meteorological parameters can be taken as constant. The implications of this inconsistency will be discussed later.

Eqs. (1), (2), (15), (16), (17), (18) and (19) constitute the complete model. We have included no equations for the net infrared exchange. This could easily be done by using an empirical relationship, such as the one proposed by Swinbank (1963) for the atmospheric emission. For these calculations, however, we have chosen to hold IR_N constant. This assumption appears to have no important effect on the model results.

The input information required to solve these equations is as follows:

1) Latitude ϕ, date, atmospheric transmission coefficient T_r, and surface albedo α.
2) U_2, T_2 and q_2, the meteorological conditions at height Z_2.
3) Z_0, the roughness of the canopy.
4) C, k_s and RH, the soil heat capacity, conductivity, and atmospheric relative humidity near the soil surface.
5) T_b, the temperature at depth $2d$.

The fundamental assumptions made in formulating this model are that 1) horizontally homogeneity is assumed in all meteorological and soil parameters; 2) the turbulent diffusivities for heat and water vapor is given by the near-neutral value for momentum; 3) the turbulent fluxes of heat and water vapor are assumed to be constant between Z_0 and Z_2; 4) the canopy is uniquely characterized by the roughness length Z_0; 5) the relative humidity at height Z_0 is a constant; 6) temperature, wind speed, and specific humidity are constant at height Z_2; 7) the canopy temperature is isothermal from the surface to height Z_0; and 8) the infrared radiation balance, IR_N, is taken as a constant.

If this model were to be applied to a plant canopy, it must be recognized that the physical and biological processes within the canopy are treated in the crudest manner possible. The vertical distributions of wind speed, temperature and specific humidity within the canopy, for instance, are known to differ considerably from the logarithmic distributions assumed here. The model is too simple to allow the calculation of interesting feedback effects of the biological processes on the canopy microclimatology. In this model the only feedback of this kind permitted is through the effect of canopy growth which could be handled as an increasing roughness length Z_0.

In applying the model to a biologically inactive urban environment where the canopy consists of buildings, some of the difficulties vanish. The RH parameter can now be interpreted as the constant fraction of the city occupied by evaporating surfaces, which may be a better assumption than holding the relative humidity constant.

The two-layer soil model adopted here eliminates much of the complexity which is actually observed in soil thermal behavior. This is not, however, an important limitation in principle since as many soil layers as desired could be added. Likewise, the assumption of constant meteorological parameters at height Z_2 is not in itself a fundamental limitation, since in principle that height could be made as high as desired. However, if Z_2 is made high enough that the meteorological parameters could be considered reasonably constant, then the assumption that the logarithmic wind law continues to hold to height Z_2 becomes very dubious. The basic problem here is that the empirical data which has allowed us to specify the turbulent diffusivities applies only near the surface of the earth. At greater heights little is known about turbulent transfer.

Micrometeorological measurements have shown conclusively that the logarithmic relationships are valid only for a restricted range of atmospheric stabilities. For sufficiently stable or unstable conditions the diffusivities are functions of stability parameters, such as the Richardson number. These functions are known empirically near the surface of the earth and could be included in more complex models.

It should be emphasized that this model was deliberately formulated to be as simple as possible. It was felt, however, that a simple model would allow insight into complicated boundary layer processes which may actually require more sophisticated treatment. An analogy could be made with the barotropic model in the history of numerical weather prediction. Although the barotropic model could in no sense be said to be an accurate representation of the real atmosphere, that model nevertheless provided enormous insight into the workings of the atmosphere as well as practical forecasts within a restricted range of conditions. We have had similar experiences with the energy budget model under discussion. It has provided insight to students as well as professional workers in micrometeorology. In addition, the model has been found to have a surprising range of practical applications. Using the model we have been able to calculate the incubation time of the African desert locust under a variety of meteorological conditions, and we have studied the body temperature of desert beetles. We are now working with a more advanced model which includes a realistic net infrared exchange calculation, Richardson number dependency for the turbulent diffusivities, a multi-layer soil calculation, and soil moisture transfer.

4. Application of the model to the urban boundary layer

The urban heat island phenomenon was brought to the author's attention by colleagues working in the area of human ecology, particularly K. E. F. Watt. The kinds of questions which arise in this context are the following. How large can the heat island effect be expected to be for the projected supercities on the east and west coasts of the United States? What are the dominant causes of the urban temperature excess? Are there engineering techniques which could be employed to reduce the heat island in existing cities? Could cities of the future be planned to eliminate the heat island? Could the processes which give rise to the urban heat island be exploited for beneficial effects in agriculture, such as frost protection? In general, these problems center around the question of whether or not the microclimate can be rationally planned and controlled.

As mentioned above, we concentrate here on boundary layer processes and will not consider the effect of pollution on incoming or outgoing radiation. In addition, all our calculations have been made for clear skies.

In applying our model to the urban boundary layer we have been forced to make a series of more or less arbitrary decisions as to the proper input data due to the rarity of information on the physical characteristics of cities. We have chosen the region in the vicinity of Davis, Calif., for a representative rural site because micrometeorological data is available from the field site maintained near there by the University of California. We have compared the calculations for this rural site with those for a nearby hypothetical city occupying 100 km² with a population of 1 million. This hypothetical city should not be identified too closely with Sacramento as that city is atypical in respect to amount of vegetation and proximity to several rivers.

We have specified the city substrate to be 20 cm of concrete, which is taken to represent the entire mass of the city. Doubtless the nature of the building and paving materials may differ considerably from this choice but proper data were unknown to the author. The temperature at a depth of 20 cm was taken as constant and given the climatological value for the month in question. Values of the roughness length for cities are available from Slade (1968). A dense array of one-story buildings is supposed to have a Z_0 of 5 ft and a two-story city a Z_0 of 10 ft. In some of our calculations we have taken the figure of 15 ft for a three-story city.

We have taken Z_2, the height at which the meteorological parameters are held constant, to be 300 m. Our argument is that although the diurnal temperature variation does certainly extend above that height this is the best compromise with the competing requirement that Z_2 be low enough that the fluxes can be considered constant with height. We have clearly satisfied neither requirement and this is taken to be part of the price we pay for a relatively simple model. We have chosen the data at 300 m from meteorological experience in the Davis-Sacramento area.

One of the interesting questions which arises in connection with urban heat budgets is the magnitude of the self-heating effect due to combustion and dissipation of other energies. Reliable figures are difficult to come by. Davidson (1967) has made estimates for New York City, but this is a very special city in terms of the mag-
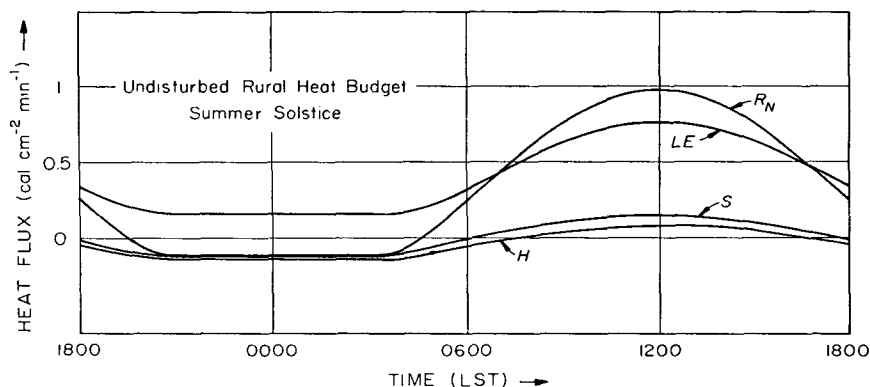
FIG. 1. The rural heat budget calculated for the summer solstice. Soil and meteorological parameters are taken for the Davis-Sacramento area.

nitude of the industrial activity and the severity of the winters. We have chosen then to estimate the heat energy released in our hypothetical city as follows. Morrison and Readling (1968) calculate that the total energy released in the United States in 1965 was 53.8×10^{15} Btu. This is equivalent to a per capita expenditure of 6.78×10^{10} cal year^{-1}. For our hypothetical city this figure reduces to an energy flux of 0.013 cal cm^{-2} min^{-1}. In order to evaluate the effect of this additional energy on city temperature we have simply added a figure of 0.05 cal cm^{-2} min^{-1} to the net radiation term in the model equations. We have increased the size of the self-heating over our calculation to allow for the fact that domestic heating would be concentrated in the winter. For the calculations which follow, most of which are for the summer solstice, the self-heating effect specified above must be a large overestimation.

The model equations can be conveniently solved by either analog or digital techniques. For classroom convenience, we have chosen to solve the present model by means of a desktop analog machine, the Electronics Associates, Inc., TR10.

5. Results

We have applied the energy budget model discussed above to a number of special cases chosen to give insight into, if not conclusively answer, some of the questions posed above. In addition we have performed a systematic sensitivity analysis of the model equations for the leading input parameters. Except for case presented in Section 5f, in which seasonal variation of the heat island is considered, all of these calculations were made for the summer solstice.

a. City vs surrounding countryside

For this calculation, which we take as the basic case from which to form variations, we have taken the city as a dense array of one-story buildings with 10% of the city occupied by evaporating areas. For the rural calculations we have used the thermal properties of the soils in the vicinity of Davis, where $k_s = 2.6 \times 10^{-3}$ cal cm^{-1} sec^{-1} (°C)$^{-1}$ and $\rho_s C = 0.52$ cal cm^{-3} (°C)$^{-1}$. The rural ground cover is assumed to be grass 10 cm high with a roughness length of 1 cm with 100% of the area
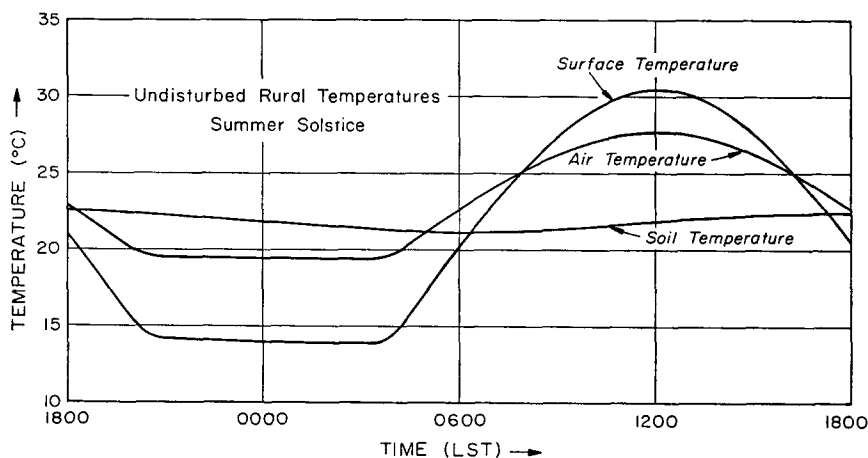

FIG. 2. Surface, air and soil temperatures calculated for a rural area in the Davis-Sacramento area for the summer solstice.
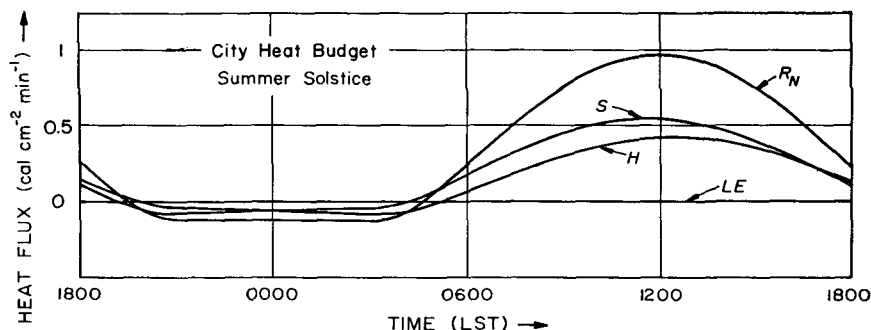
FIG. 3. The surface heat budget calculated for a hypothetical city in the vicinity of Davis, Calif., for the summer solstice.

being taken as occupied by freely transpiring plants. For both city and country we have taken the same values for the following parameters. The albedo is 20%, the transmission coefficient is 0.664, the temperature at 20 cm below the surface is 21.3C (the climatological mean for June), the wind speed, temperature and specific humidity at 300 m are 5 m sec^{-1}, 21.3C and 5×10^{-3}, respectively, and the net infrared balance is held constant at 0.1 cal cm^{-2} min^{-1}.

The results are shown in Figs. 1 and 2. The rural heat budget, shown in Fig. 1, indicates that the majority of the available energy is going into evaporating water. The general nature of the energy budget during the day, including the magnitude of the net radiation, agrees with measured heat budgets at the Davis site (Pruitt et al., 1967). At night, the model overpredicts evaporation. It is interesting that the model correctly estimates evapotranspiration during the day, although the diffusivity used in these calculations is equivalent to the Thornthwaite-Holzman (1939) evaporation formula, which is known to underestimate evaporation during unstable conditions. Evidently, the additional requirement that the energy balance be satisfied improves this approach. The corresponding rural temperatures for the soil at a 10 cm depth, the surface, and air at a height of 5 ft are shown in Fig. 2. The air temperature rises from a minimum of 19.3C to a maximum of 27.7C. These are reasonable values according to local experience although the minimum is somewhat high.

Figs. 3 and 4 show the companion calculations made for the city. The heat budget, shown in Fig. 3, includes the same net radiation as for the rural case. This energy, however, is used much differently. Most of the energy goes into heating the concrete. No energy appears in the latent heat flux term because with only a 10% evaporating area in the city, the specific humidity at street level does not reach high enough values to cause an upward diffusion of water vapor. The corresponding air temperature at 5 ft and concrete temperature at a depth of 10 cm are shown in Fig. 4. The air temperature reaches a maximum of 33.7C after a minimum of 22.7C. The model thus produces a heat island of 6.0C in the maximum temperature and 4.1C in the minimum. Although the model predicts the largest temperature excess during the day rather than at night, as is observed, this result seems encouraging enought to call for a more detailed analysis, which we present in the following cases.

b. City park case

Since the largest heat island effect reported in the literature was for a city park (Duckworth and Sandberg, 1954), we have made a special calculation for this case. Here we have replaced the city concrete with soil characteristic of the surrounding countryside, a 3.4 m canopy of trees with a roughness length of 200 cm, and specified 90% of the area of the park was freely trans-
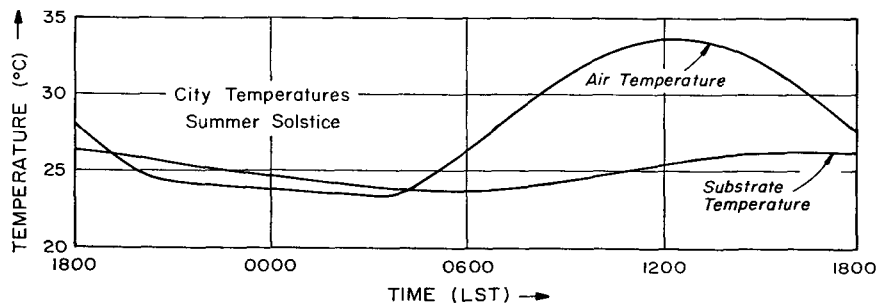

FIG. 4. Air and substrate temperature calculated for a hypothetical city in the vicinity of Davis, Calif., for the summer solstice.
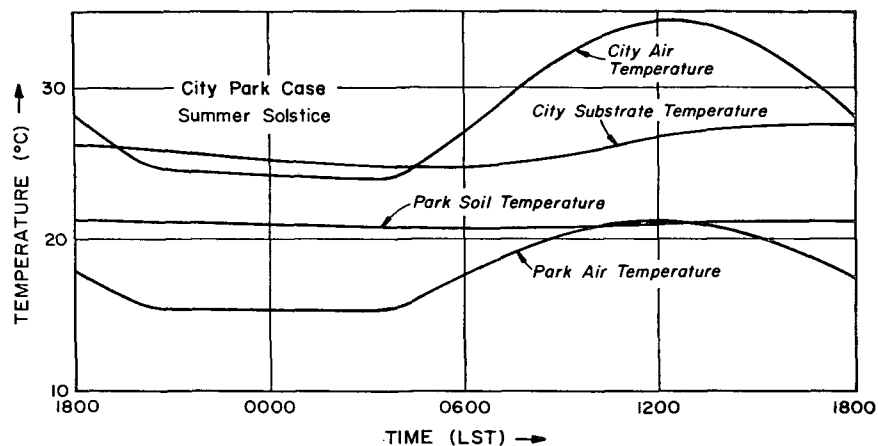
Fig. 5. Air and soil temperature for a city park area calculated for the summer solstice.
Temperatures calculated for the city center are also shown for comparison.

piring. All other parameters are the same as for the city case. Fig. 5 shows the soil and air temperature at 5 ft for the park. Also shown for comparison are the corresponding city temperatures. The calculation indicates the park air temperature to be 11.5C cooler at midday and 8.1C at dawn. These figures are close to those observed by Duckworth and Sandberg, although again the largest excess is calculated to be during the day rather than night.

c. Self-heating case

For this case, we have added a constant 0.05 cal cm^{-2} min^{-1} heat flux to the net radiation to simulate heat released within the city due to the activities of men. As we indicated above, this figure is probably much too high for the summer solstice. All other parameters were specified to be the same as for the city case already discussed. The result of this calculation showed the city air temperature to be raised by 0.8C due to the self-heating mechanism. There was no diurnal variation in the magnitude of this effect.

d. Moist city vs dry countryside

In many agricultural areas crops are not irrigated. Under these conditions the specification that 100% of

the area is freely transpiring may be too high. In addition, Kratzer (1956) lists large European cities with up to 24% of their area occupied by woods. For smaller cities this figure rises to 38%. Consequently, we have repeated the city vs countryside study, holding all parameters the same as before except that we now specify that 50% of the countryside and 30% of the city is occupied by transpiring plants. Under these conditions we obtain maximum air temperatures at 5 ft of 32.3C in the country and 29.4C in the city. In this case the city is a "cool" rather than heat island during the day. At night the situation is reversed, the minimum temperature is 22.4C in the city and 22.1C in the country. Presumably, these results are related to observations that some cities are cooler than their surroundings during the day. Again it appears that the night temperature in this model is too high.

e. Transverse from city to country

In order to see more clearly how the rural climate relates and merges into the city, we have calculated a traverse from the edge of the urban area into the city center. For these calculations we have varied the conductivity and diffusivity of the underlying surface, the roughness length, the evaporating area, and held all other parameters to be the same as the first case considered. The values used are shown in Table 1. Fig. 6 shows the maximum and minimum temperatures calculated at various distances from the city center for the summer solstice. It can be seen that the highest temperature is found at a distance of 1 km from the city center. This occurs because the increased roughness at the city center (three-story buildings assumed) results in more efficient diffusion of heat upward while the model treats the 10% evaporating area at 1 km as no evaporation so no energy is absorbed in the latent heat mechanism in the city center. The largest rate of change of the maximum temperature occurs between 2 and 3

TABLE 1. Input data for city to country traverse.

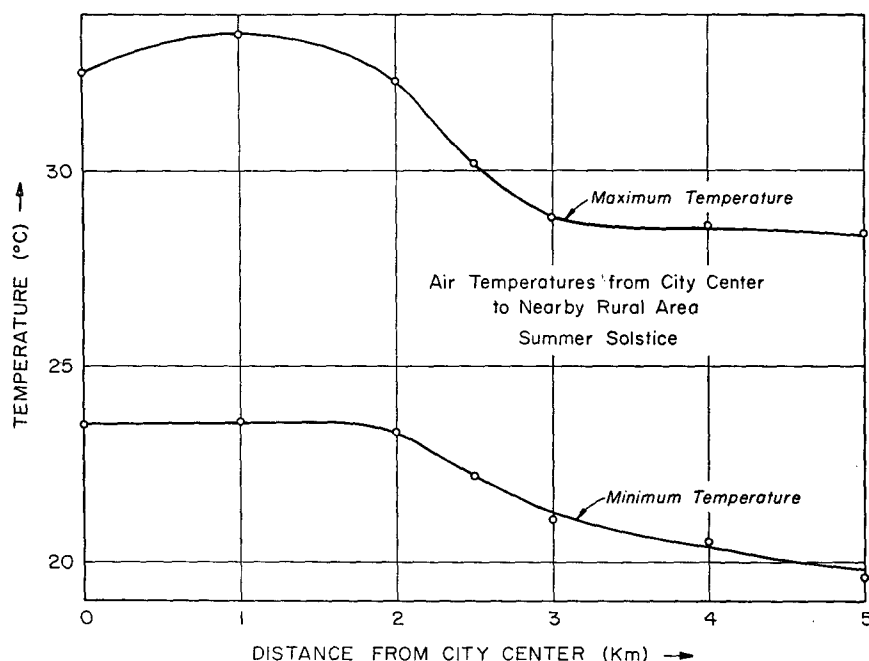| Distance from city center (km)	Conductivity [cal cm^{-1} sec^{-1} (°C)$^{-1}$]	Diffusivity (cm^2 sec^{-1})	z_0 (cm)	Evaporating area (%)
0	11.00×10^{-3}	20 ×10^{-3}	305	0
1	9.33×10^{-3}	17 ×10^{-3}	250	10
2	7.65×10^{-3}	14 ×10^{-3}	200	20
2.5	6.80×10^{-3}	12.5×10^{-3}	150	30
3	5.97×10^{-3}	11 ×10^{-3}	100	40
4	4.28×10^{-3}	8 ×10^{-3}	10	60
5	2.60×10^{-3}	5 ×10^{-3}	1	90

FIG. 6. Air temperature calculated at various distances from the city center for the summer solstice. The various meteorological and substrate parameters chosen for this calculation are given in Table 1.

km. As we shall see in the sensitivity analysis to be presented below, this seems to be mainly due to the change in evaporating area, as the model is most sensitive to this parameter in the vicinity of a value of 25%.

f. Seasonal variation of the heat island

In order to investigate the seasonal variation in the heat island phenomenon, we have repeated the calculation at the times of the spring and fall equinoxes and for the winter solstice. As much as was possible with the data available, we have used climatological values for the meteorological parameters at height Z_2 and the soil boundary condition. Otherwise, the input information was the same as for the first case discussed. The calculated temperatures are shown in Table 2, where ΔT is temperature difference between city and country. It can be seen, according to the calculation, that the city is always the warmest with the largest excess found at

TABLE 2. Seasonal variation of the heat island.

Date	T_{max} (°C) City	Country	ΔT	T_{min} (°C) City	Country	ΔT
Spring equinox	17.2	15.7	1.5	10.8	8.2	2.6
Summer solstice	33.7	27.7	6.0	23.4	19.3	4.1
Fall equinox	27.9	24.0	3.9	21.0	17.4	3.6
Winter solstice	13.5	12.5	1.0	9.2	6.7	2.5

the times of the summer solstice and fall equinox and the smallest at the winter solstice. It is interesting that the model predicts the night heat island to be larger than the day heat island during the winter and spring.

g. Sensitivity analysis

In order to see more clearly how the various competing parameters combine to produce the calculated temperatures, we have made a systematic sensitivity analysis for the leading parameters. The strategy has been to use the summer solstice city case presented in Section 5a as the base calculation, and to repeat the city temperature calculation for various values of a particular parameter while holding all others constant. In this way we have investigated the effect of wind speed, albedo, roughness and evaporation on the city temperature. Table 3 and Fig. 7 show the result for wind speed. It can be seen that city temperatures are most sensitive to wind speed at low wind speeds and that increasing wind speed has the effect of decreasing the

TABLE 3. Effect of wind speed on city temperature.

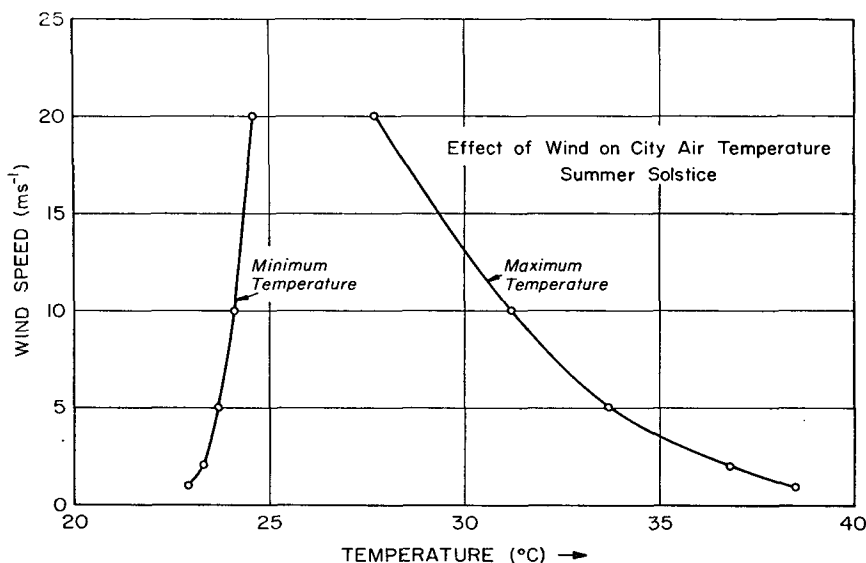| U_2 (m sec^{-1})	T_{max} (°C)	T_{min} (°C)	Sensitivity [°C (m sec^{-1})$^{-1}$] Day	Night	Wind range (m sec^{-1})
1	38.5	22.9	−1.70	+0.40	1–2
2	36.8	23.3	−1.03	+0.13	2–5
5	33.7	23.7	−0.50	+0.08	5–10
10	31.2	24.1	−0.35	+0.05	10–20
20	27.7	24.6			

FIG. 7. The effect of wind speed on summer solstice air temperature
in the city according to the model.

maximum and increasing the minimum temperature. Fig. 8 is a tracing of the actual machine output from which the above figures were obtained. The calculation was continued 5 days to make sure that equilibrium conditions were obtained, since the first day's temperature was sensitive to the particular initial temperature chosen at a depth of 10 cm. In each of the following sensitivity cases, the same procedure was followed to ensure equilibrium values.

Table 4 shows the results of the albedo calculation. The albedo is calculated to have relatively small effect on city temperature over this albedo range, which seems appropriate for cities according to the available information. The overall sensitivity is 1.25C per $\frac{1}{10}$ albedo change.

The effect of roughness on the summer solstice city temperature is summarized in Table 5.

It is seen that increasing the roughness length from values appropriate for grass to those for two- and three-story buildings decreases the maximum temperature considerably while slightly increasing the minimum temperature. When this data is plotted on semi-log paper it is not linear but a reasonable overall sensitivity figure for the maximum temperature would be −2.3C per decade increase in Z_0.

The effect of increasing the fraction of the city from which evapotranspiration is occurring is shown in Fig. 9 and Table 6.

As we anticipated above, the city temperature is most sensitive to evaporation in the case when the evaporating area is between 0.2 and 0.3. Here the effect of increasing the evaporating fraction by 0.1 is to decrease the maximum temperature by 3.5C.

Table 7 shows the effect of changing the temperature at height Z_2, 300 m in this case, on the city temperature at 5 ft.

The overall sensitivity was 0.59C $(°C)^{-1}$ for the maximum and 0.61C $(°C)^{-1}$ for the minimum temperature.

6. Discussion and conclusions

The general nature of the calculations presented above appear to be quite encouraging. Although the energy budget model is relatively crude, these calculations are qualitatively and quantitatively reasonable. The energy budget approach appears to explain many of the observed features of the urban heat island phenomenon.

TABLE 4. Effect of albedo on city temperature.

Albedo	T_{max} (°C)	T_{min} (°C)
0.10	35.2	23.7
0.15	34.4	23.7
0.20	33.7	23.7
0.25	33.3	23.6
0.30	32.7	23.6

TABLE 5. Effect of roughness length on city temperature.

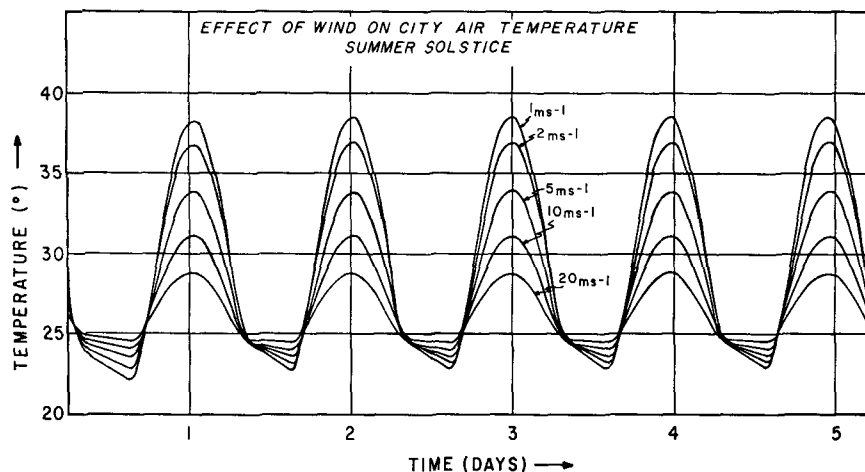| z_0 (cm)	T_{max} (°C)	T_{min} (°C)
1	37.9	23.0
10	36.7	23.2
100	34.4	23.4
152	33.7	23.6
200	33.4	23.6
250	33.0	23.7
305	32.7	23.7
457	31.9	23.8

FIG. 8. Tracing of the machine output from which Fig. 7 was prepared.

Our results indicate that the urban temperature excess is the net effect of several competing physical processes, each of which, if acting alone, could produce relatively large temperature contrasts. Generally, there is a tendency for a cancellation effect so that ordinarily the temperature contrast between city and country is small. For instance, the decrease in evaporation as the city center is approached is balanced by the increasing size of the buildings which augments the diffusion of heat upward. In the city park case these two factors work together to produce the largest contrast with the city center.

In general, our model indicates that the most important parameters which determine the size of the heat island effect are the reduction of evaporation in the city, the increased roughness of the city, the thermal properties of the building and paving materials, and wind speed. Overall, reduced evaporation dominates during the day while the thermal properties of the substrate dominate at night, other factors being equal. Our calculations indicate that self-heating is probably not an important contributor to the heat island, although it may be detectable in some circumstances.

Probably the outstanding failure of this model is the fact, for the summer solstice and fall equinox, that the heat island is calculated to be largest during the day while all observations indicate that the temperature excess is actually largest at night. Calculations with a more sophisticated model will be required to identify the cause of this discrepancy. Two possibilities would be the radiation blanket effect of pollution or Richardson number effects on the turbulent diffusivities at night.

In the future, we plan to make further calculations for the urban boundary layer and other problems with a more advanced model. The improvement to be expected, however, is limited by the nature of the data available. Information as to the physical nature of cities, such as the average thermal properties or the total green area, and roughness length of cities, is hard to come by. It would seem that the gathering of such information would be an excellent objective for the remote sensing technology.

Finally, the relative success of this simple model in predicting city temperature as a function of boundary layer parameters raises the possibility of the engineering use of such models in city planning. It seems highly

TABLE 6. Effect of evaporation on city temperature.

Evapo-rating fraction	T_{max} (°C)	T_{min} (°C)	Sensitivity [°C (0.1 change)$^{-1}$] Day	Night	Fractional range
0.0	34.6	24.3	0.0	0.0	0–0.1
0.1	34.6	24.3	−2.2	−0.2	0.1–0.2
0.2	32.4	24.1	−3.5	−1.3	0.2–0.3
0.3	29.9	22.8	−2.1	−1.4	0.3–0.4
0.4	27.8	21.4	−1.6	−1.1	0.4–0.5
0.5	26.2	20.3			

TABLE 7. Effect of temperature T_2 at 300 m on city temperature.

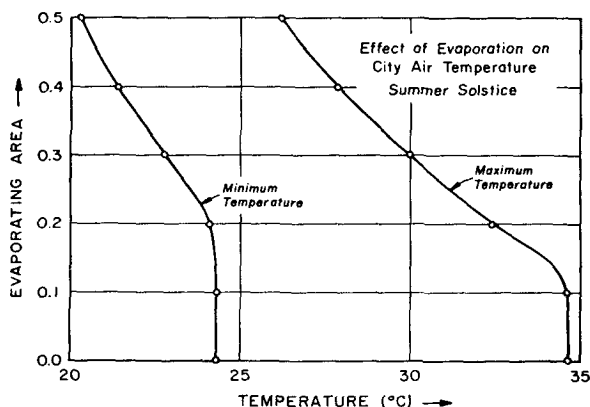| T_2 (°C)	T_{max} (°C)	T_{min} (°C)
15	29.7	19.6
17	31.0	20.8
19	32.3	22.2
21	33.4	23.3
23	34.3	24.2
25	35.7	25.7
27	36.8	26.8
29	38.0	28.2

FIG. 9. The effect of evaporation on city air temperature on the summer solstice according to the model.

desirable that cities be planned to eliminate the sweltering summer nights often experienced in eastern cities, for instance. According to our calculations, the addition of sufficient plants would improve the summer climate of existing cities, particularly if the green area is near the critical 20% value. Perhaps roof-top parks would be a feasible way in which to modify existing cities.

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